drives in high precision turntables and hobbing machines. **Figure 14-12** presents the basic concept of a duplex lead worm.

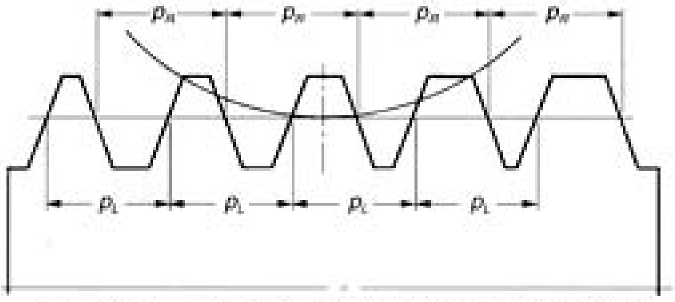


Fig. 14-12 Basic Concepts of Duplex Lead Worm

The lead or pitch, P_L and P_R, on the two sides of the worm thread are not identical. The example in **Figure 14-12** shows the case when $P_R > P_L$ To produce such a worm requires a special dual lead hob.

The intent of **Figure 14-12** is to indicate that the worm tooth thickness is progressively bigger towards the right end. Thus, it is convenient to adjust backlash by simply moving the duplex worm in the axial direction.

SECTION 15 GEAR ACCURACY

Gears are one of the basic elements used to transmit power and position. As designers, we desire them to meet various demands:

1. Minimum size.
2. Maximum power capability.
3. Minimum noise (silent operation).
4. Accurate rotation/position.

To meet various levels of these demands requires appropriate degrees of gear accuracy. This involves several gear features.

15.1 Accuracy Of Spur And Helical Gears

This discussion of spur and helical gear accuracy is based upon JIS B 1702 standard. This specification describes 9 grades of gear accuracy - grouped from 0 through 8 - and four types of pitch errors:

Single pitch error.
Pitch variation error.
Accumulated pitch error.
Normal pitch error.

Single pitch error, pitch variation and accumulated pitch errors are closely related with each other.

15.1.1 Pitch Errors of Gear Teeth

1. Single Pitch Error (f_{pt})

The deviation between actual measured pitch value between any adjacent tooth surface and theoretical circular pitch.

2. Pitch Variation Error (f_{pu})

Actual pitch variation between any two adjacent teeth. In the ideal case, the pitch variation error will be zero.

3. Accumulated Pitch Error (F_p)

Difference between theoretical summation over any number of teeth interval, and summation of actual pitch measurement over the same interval.

4. Normal Pitch Error (f_{pb})

It is the difference between theoretical normal pitch and its actual measured value.

The major element to influence the pitch errors is the runout of gear flank groove.

Table 15-1 contains the ranges of allowable pitch errors of spur gears and helical gears for each precision grade, as specified in JIS B 1702-1976.

Table 15-1 The Allowable Single Pitch Error, Accumulated Pitch Error and Normal Pitch Error, μm

Grade	Single Pitch Error f_{pt}	Accumulated Pitch Error F_p	Normal Pitch Error f_{pb}
JIS 0	$0.5W + 1.4$	$2.0W + 5.6$	$0.9W' + 1.4$
1	$0.71W + 2.0$	$2.8W + 8.0$	$1.25W' + 2.0$
2	$1.0W + 2.8$	$4.0W + 11.2$	$1.8W' + 2.8$
3	$1.4W + 4.0$	$5.6W + 16.0$	$2.5W' + 4.0$
4	$2.0W + 5.6$	$8.0W + 22.4$	$4.0W' + 6.3$
5	$2.8W + 8.0$	$11.2W + 31.5$	$6.3W' + 10.0$
6	$4.0W + 11.2$	$16.0W + 45.0$	$10.0W' + 16.0$
7	$8.0W + 22.4$	$32.0W + 90.0$	$20.0W' + 32.0$
8	$16.0W + 45.0$	$64.0W + 180.0$	$40.0W' + 64.0$

In the above table, W and W' are the tolerance units defined as:

$$W = \sqrt[3]{d} + 0.65m \ (\mu m) \qquad \textbf{(15-1)}$$

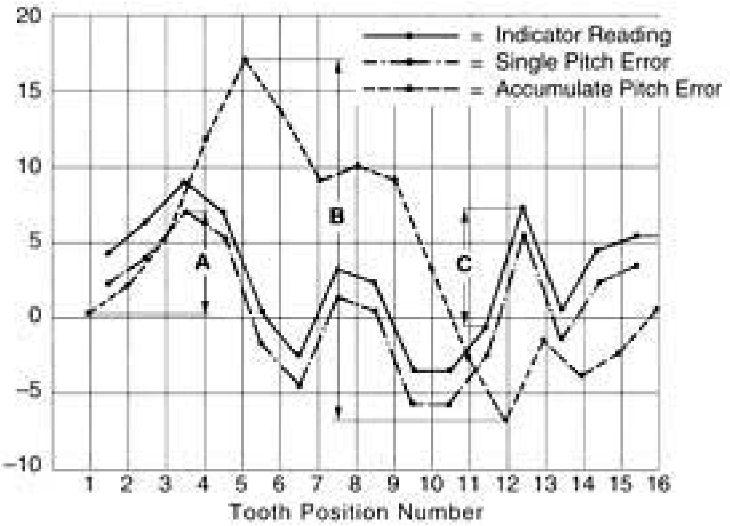
$$W = 0.56\ W + 0.25m\ (\mu m) \qquad \textbf{(15-2)}$$

The value of allowable pitch variation error is k times the single pitch error. **Table 15-2** expresses the formula of the allowable pitch variation error.

Table 15-2 The Allowable Pitch Variation Error, μm

Single Pitch Error, f_{pt}	Pitch Variation Error, f_{pu}
less than 5	$1.00f_{pt}$
5 or more, but less than 10	$1.06f_{pt}$
10 or more, but less than 20	$1.12f_{pt}$
20 or more, but less than 30	$1.18f_{pt}$
30 or more, but less than 50	$1.25f_{pt}$
50 or more, but less than 70	$1.32f_{pt}$
70 or more, but less than 100	$1.40f_{pt}$
100 or more, but less than 150	$1.50f_{pt}$
more than 150	$1.60f_{pt}$

Figure 15-1 is an example of pitch errors derived from data measurements made with a dial indicator on a 15 tooth gear. Pitch differences were measured between adjacent teeth and are plotted in the figure. From that plot, single pitch, pitch variation and accumulated pitch errors are extracted and plotted.



NOTE: A = Max. Single Pitch Error
B = Max. Accumulated Error
C = Max. Pitch Variation Error

Fig. 15-1 Examples of Pitch Errors for a 15 Tooth Gear

15.1.2 Tooth Profile Error, f_f

Tooth profile error is the summation of deviation between actual tooth profile and correct involute curve which passes through the pitch point measured perpendicular to the actual profile. The measured band is the actual effective working surface of the gear. However, the tooth modification area is not considered as part of profile error.

15.1.3 Runout Error of Gear Teeth, F_r

This error defines the runout of the pitch circle. It is the error in radial position of the teeth. Most often it is measured by indicating the position of a pin or ball inserted in each tooth space around the gear and taking the largest difference. Alternately, particularly for fine pitch gears, the gear is rolled with a master gear on a variable center distance fixture, which records the change in the center distance as the measure of teeth or pitch circle runout. Runout causes a number of problems, one of which is noise. The source of this error is most often insufficient accuracy and ruggedness of the cutting arbor and tooling system.

15.1.4 Lead Error, f_β

Lead error is the deviation of the actual advance of the tooth profile from the ideal value or position. Lead error results in poor tooth contact, particularly concentrating contact to the tip area. Modifications, such as tooth crowning and relieving can alleviate this error to some degree.

Shown in **Figure 15-2** is an example of a chart measuring tooth profile error and lead error using a Zeiss UMC 550 tester.

Table 15-3 presents the allowable tooth profile, runout and lead errors per JIS B 1702-1976.

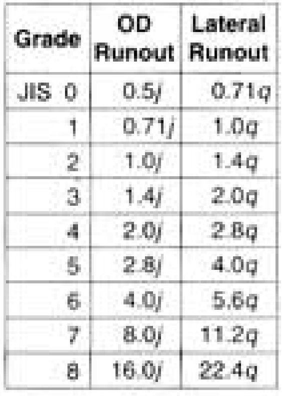
15.1.5. Outside Diameter Runout and Lateral Runout

To produce a high precision gear requires starting with an accurate gear blank. Two criteria are very important:

1. Outside diameter (OD) runout.
2. Lateral (side face) runout.

The lateral runout has a large impact on the gear tooth accuracy. Generally, the permissible runout error is related to the gear size. **Table 15-4** presents equations for allowable values of OD runout and lateral runout.

Table 15-4 The Value of Allowable OD and Lateral Runout, μm



Grade	OD Runout	Lateral Runout
JIS 0	$0.5j$	$0.71q$
1	$0.71j$	$1.0q$
2	$1.0j$	$1.4q$
3	$1.4j$	$2.0q$
4	$2.0j$	$2.8q$
5	$2.8j$	$4.0q$
6	$4.0j$	$5.6q$
7	$8.0j$	$11.2q$
8	$16.0j$	$22.4q$

where: $j = 1.1\sqrt[3]{d_a} + 5.5$

d_a = Outside diameter (mm)

$q = \frac{6d}{b + 50} + 3$

d = Pitch diameter (mm)

b = Tooth width (mm)

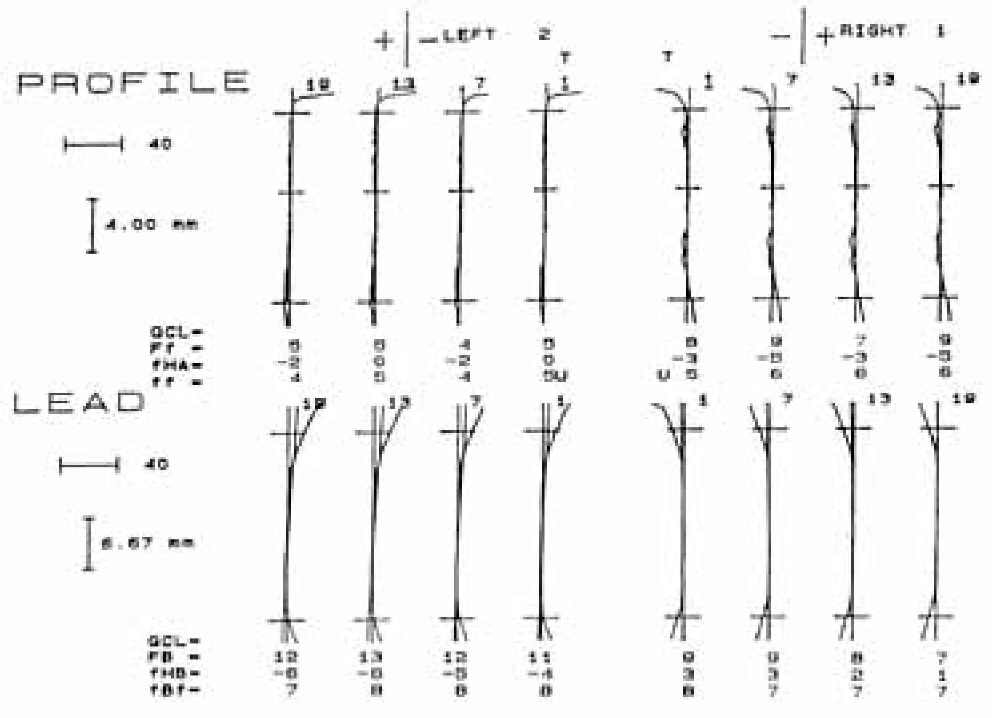


Fig. 15-2 A Sample Chart of Profile and Lead Error Measurement

Table 15-3 The Value of Allowable Tooth Profile Error, Runout Error and Lead Error, μm

Grade	Tooth Profile Error f_f	Runout Error of Gear Groove F_r	Lead Error F_β
JIS 0	0.71m + 2.24	1.4W + 4.0	0.63(0.1b + 10)
1	1.0m + 3.15	2.0W + 5.6	0.71(0.1b + 10)
2	1.4m + 4.5	2.8W + 8.0	0.80(0.1b + 10)
3	2.0m + 6.3	4.0W + 11.2	1.00(0.1b + 10)
4	2.8m + 9.0	5.6W + 16.0	1.25(0.1b + 10)
5	4.0m + 12.5	8.0W + 22.4	1.60(0.1b + 10)
6	5.6m + 18.0	11.2W + 31.5	2.00(0.1b + 10)
7	8.0m + 25.0	22.4W + 63.0	2.50(0.1b + 10)
8	11.2m + 35.5	45.0W + 125.0	3.15(0.1b + 10)

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm)
b = Tooth width (mm)
m = Module (mm)

15.2 Accuracy Of Bevel Gears

JIS B 1704 regulates the specification of a bevel gears accuracy. It also groups bevel gears into 9 grades, from 0 to 8.

There are 4 types of allowable errors:

1. Single Pitch Error.
2. Pitch Variation Error.
3. Accumulated Pitch Error.
4. Runout Error of Teeth (pitch circle).

These are similar to the spur gear errors.

1. Single Pitch Error, f_{pt}

The deviation between actual measured pitch value between any adjacent teeth and the theoretical circular pitch at the central cone distance.

2. Pitch Variation Error, f_{pu}

Absolute pitch variation between any two adjacent teeth at the central cone distance.

3. Accumulated Pitch Error, F_p

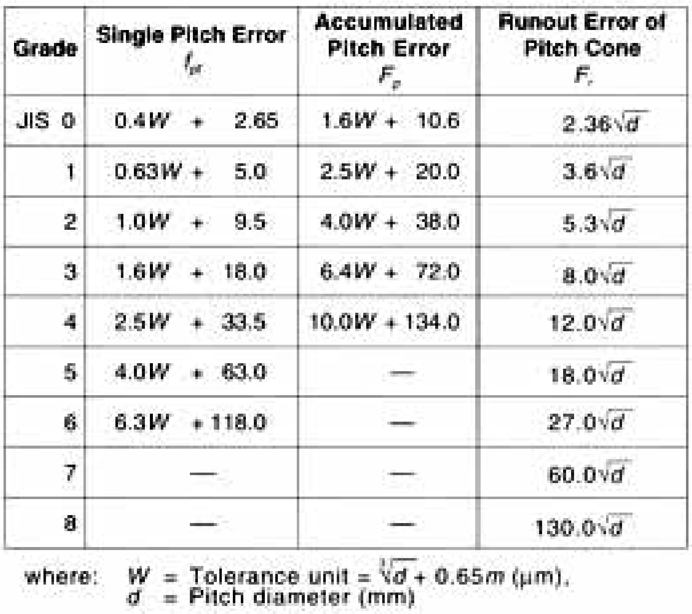
Difference between theoretical pitch sum of any teeth interval, and the summation of actual measured pitches for the same teeth interval at the central cone distance.

4. Runout Error of Teeth, F_r

This is the maximum amount of tooth runout in the radial direction, measured by indicating a pin or ball placed between two teeth at the central cone distance. It is the pitch cone runout.

Table 15-5 presents equations for allowable values of these various errors.

Table 15-5 Equations for Allowable Single Pitch Error, Accumulated Pitch Error and Pitch Cone Runout Error, μm



Grade	Single Pitch Error f_{pt}	Accumulated Pitch Error F_p	Runout Error of Pitch Cone F_r
JIS 0	$0.4W + 2.65$	$1.6W + 10.6$	$2.36\sqrt{d}$
1	$0.63W + 5.0$	$2.5W + 20.0$	$3.6\sqrt{d}$
2	$1.0W + 9.5$	$4.0W + 38.0$	$5.3\sqrt{d}$
3	$1.6W + 18.0$	$6.4W + 72.0$	$8.0\sqrt{d}$
4	$2.5W + 33.5$	$10.0W + 134.0$	$12.0\sqrt{d}$
5	$4.0W + 63.0$	—	$18.0\sqrt{d}$
6	$6.3W + 118.0$	—	$27.0\sqrt{d}$
7	—	—	$60.0\sqrt{d}$
8	—	—	$130.0\sqrt{d}$

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm),
d = Pitch diameter (mm)

Table 15-6 The Formula of Allowable Pitch Variation Error (μm)

Single Pitch Error, fpt	**Pitch Variation Error, f**pu
Less than 70	1.3fpt
70 or more, but less than 100	1.4fpt
100 or more, but less than 150	1.5fpt
More than 150	1.6fpt

The equations of allowable pitch variations are in **Table 15-6**.

Besides the above errors, there are seven specifications for bevel gear blank dimensions and angles, plus an eighth that concerns the cut gear set:

1. The tolerance of the blank outside diameter and the crown to back surface distance.
2. The tolerance of the outer cone angle of the gear blank.
3. The tolerance of the cone surface runout of the gear blank.
4. The tolerance of the side surface runout of the gear blank.
5. The feeler gauge size to check the flatness of blank back surface.
6. The tolerance of the shaft runout of the gear blank.
7. The tolerance of the shaft bore dimension deviation of the gear blank.
8. The contact band of the tooth mesh.

Item 8 relates to cutting of the two mating gears' teeth. The meshing tooth contact area must be full and even across the profiles. This is an important criterion that supersedes all other blank requirements.

15.3 Running (Dynamic) Gear Testing

An alternate simple means of testing the general accuracy of a gear is to rotate it with a mate, preferably of known high quality, and measure characteristics during rotation. This kind of tester can be either single contact (fixed center distance method) or dual (variable center distance method). This refers to action on one side or simultaneously on both sides of the tooth. This is also commonly referred to as single and double flank testing. Because of simplicity, dual contact testing is more popular than single contact. JGMA has a specification on accuracy of running tests.

1. Dual Contact (Double Flank) Testing

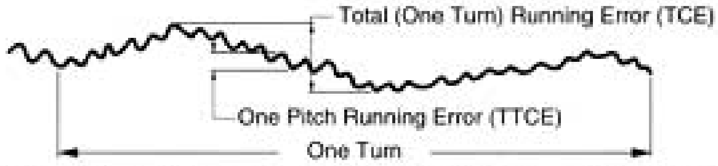


Fig. 15-3 Example of Dual Contact Running Testing Report

In this technique, the gear is forced meshed with a master gear such that there is intimate tooth contact on both sides and, therefore, no backlash. The contact is forced by a loading spring. As the gears rotate, there is variation of center distance due to various errors, most notably runout. This variation is measured and is a criterion of gear quality. A full rotation presents the total gear error, while rotation through one pitch is a tooth-to-tooth error. **Figure 15-3** presents a typical plot for such a test.

For American engineers, this measurement test is identical to what AGMA designates as Total Composite Tolerance (or error) and Tooth-to-Tooth Composite Tolerance. Both of these parameters are also referred to in American publications as "errors", which they truly are. Tolerance is a design value which is an inaccurate description of the parameter, since it is an error.

Allowable errors per JGMA 116-01 are presented on the next page, in **Table 15-7**.

2. Single Contact Testing

In this test, the gear is mated with a master gear on a fixed center distance and set in such a way that only one tooth side makes contact. The gears are rotated through this single flank contact action, and the angular transmission error of the driven gear is measured. This is a tedious testing method and is seldom used except for inspectiop of the very highest precision gears.

Table 15-7 Allowable Values of Running Errors, mm

Grade	Tooth-to-Tooth Composite Error	Total Composite Error
0	1.12m + 3.55	(1.4W + 4.0) + 0.5(1.12m + 3.55)
1	1.6m + 5.0	(2.0W + 5.6) + 0.5(1.6m + 5.0)
2	2.24m + 7.1	(2.8W + 8.0) + 0.5(2.24m + 7.1)
3	3.15m + 10.0	(4.0W + 11.2) + 0.5(3.15m + 10.0)
4	4.5m + 14.0	(5.6W + 16.0) + 0.5(4.5m + 14.0)
5	6.3m + 20.0	(8.0W + 22.4) + 0.5(6.3m + 20.0)
6	9.0m + 28.0	(11.2W + 31.5) + 0.5(9.0m + 28.0)
7	12.5m + 40.0	(22.4W + 63.0) + 0.5(12.5m + 40.0)
8	18.0m + 56.0	(45.0W + 125.0) + 0.5(18.0m + 56.0)

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm)
d = Pitch diameter (mm)
m = Module

SECTION 16 GEAR FORCES

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shaft, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank.

16.1 Forces In A Spur Gear Mesh

The spur gear's transmission force F_n, which is normal to the tooth surface, as in **Figure 16-1**, can be resolved into a tangential component, F_u, and a radial component, F_r Refer to **Equation (16-1)**.

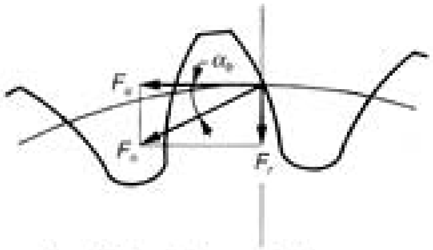


Fig. 16-1 Forces Acting on a Spur Gear Mesh

$$\left.\begin{aligned} F_u &= F_n \cos\alpha_b \\ F_r &= F_n \sin\alpha_b \end{aligned}\right\} \qquad (16\text{-}1)$$

The direction of the forces acting on the gears are shown in **Figure 16-2**. The tangential component of the drive gear, F_{u1}, is equal to the driven gear's tangential component, F_{u2}, but the directions are opposite. Similarly, the same is true of the radial components.

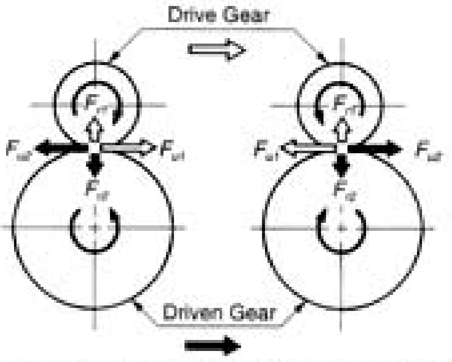


Fig. 16-2 Directions of Forces Acting on a Spur Gear Mesh

16.2 Forces In A Helical Gear Mesh

The helical gear's transmission force, F_n, which is normal to the tooth surfaces, can be resolved into a tangential component, F_1, and a radial component, F_r, as shown in **Figure 16-3**.

Table 16-1 Forces Acting Upon a Gear

Types of Gears		Tangential Force, F_u	Axial Force, F_a	Radial Force, f_r
Spur Gear		$F_u = \frac{2000\,T}{d}$	———	$F_u \tan\alpha$
Helical Gear			$F_u \tan\beta$	$F_u \frac{\tan\alpha_n}{\cos\beta}$
Straight Bevel Gear			When convex surface is working:	
Spiral Bevel Gear		$F_u = \frac{2000\,T}{d_m}$ d_m is the central pitch diameter $d_m = d - b\sin\delta$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$
			When concave surface is working:	
			$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$
Worm drive	Worm (Driver)	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$	$Fu \frac{\sin\alpha_n}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$
	Wheel (Driven)	$F_u \frac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$	F_u	
Screw Gear ($\Sigma = 90^\circ$) $\beta = 45^\circ$	Driver gear	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	$Fu \frac{\sin\alpha_n}{\cos\alpha_n\cos\beta + \mu\sin\beta}$
	Driven Gear	$F_u \frac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	F_u	