Variables, Definitions, Assumptions

Angle = α
Angle of perpindicular-to-tangent line = $\tau = 90 - \alpha/2$
Slope of perpindicular-to-tangent line = $s = tan(\tau)$

Base circle radius = r_b
Base circle center is located at $(0, 0)$
Location of tangent point on base circle = $(b_x, b_y) = (r_b * cos(\tau), r_b * sin(\tau))$

Lift = l

Nose circle radius = r_n
Nose circle center is located on the vertical center line above the base circle center
Location of nose circle center = (n_x, n_y)

Unknowns:

Location of flank circle center = (x, y)
Radius of flank circle = r

Equivalencies:

$y = sx$ (because the center of the flank circle is located on the perpindicular-to-tangent line with slope s)
$b_y = sb_x$ (because the tangent point is located on the perpindicular-to-tangent line with slope s)
$n_x = 0$ (because the nose circle is on the vertical center line)
$n_y = r_b + l - r_n$

Useful constants:

Let $k = (1+s^2)$
Let $u = 2n_ys - 2kb_x = 2(n_ys - kb_x)$
Let $v = kb_x^2 - n_y^2 - r_n^2$

Radius of flank circle, *r*

$$r = \sqrt{(x-b_x)^2 + (y-b_y)^2}$$

$$= \sqrt{(x-b_x)^2 + (sx-sb_x)^2}$$

$$= \sqrt{(x^2 - 2b_x x + b_x^2) + (s^2x^2 - 2s^2 b_x x + s^2 b_x^2)}$$

$$= \sqrt{(x^2 + s^2x^2) + (-2b_x x - 2s^2 b_x x) + (b_x^2 + s^2 b_x^2)}$$

$$= \sqrt{x^2(1+s^2) - 2b_x x(1+s^2) + b_x^2(1+s^2)}$$

$$= \sqrt{(1+s^2)(x^2 - 2b_x x + b_x^2)}$$

$$= \sqrt{k(x^2 - 2b_x x + b_x^2)}$$

$$= \sqrt{kx^2 - 2kb_x x + kb_x^2}$$

or $= \sqrt{k(x-b_x)^2}$

or $= \sqrt{k}(x-b_x)$

Distance from flank circle center *(x, y)* to nose circle center *(n_x, n_y)*, *d_n*

$$d_n = \sqrt{(x-n_x)^2 + (y-n_y)^2}$$

$$= \sqrt{(x-0)^2 + (sx-n_y)^2}$$

$$= \sqrt{x^2 + (sx-n_y)^2}$$

$$= \sqrt{x^2 + s^2x^2 - 2n_y sx + n_y^2}$$

$$= \sqrt{(1+s^2)x^2 - 2n_y sx + n_y^2}$$

$$= \sqrt{kx^2 - 2n_y sx + n_y^2}$$

If flank circle is tangent to nose circle, then $r = d_n + r_n$

$$r=\sqrt{kx^2-2kb_x x+kb_x^2}$$

$$d_n=\sqrt{kx^2-2n_y sx+n_y^2} \quad -$$

$$\sqrt{kx^2-2kb_x x+kb_x^2}=\sqrt{kx^2-2n_y sx+n_y^2}+r_n$$

$$kx^2-2kb_x x+kb_x^2=kx^2-2n_y sx+n_y^2+2r_n\sqrt{kx^2-2n_y sx+n_y^2}+r_n^2$$

$$kx^2-2kb_x x+kb_x^2-kx^2+2n_y sx-n_y^2-r_n^2=2r_n\sqrt{kx^2-2n_y sx+n_y^2}$$

$$2n_y sx-2kb_x x+kb_x^2-n_y^2-r_n^2=2r_n\sqrt{kx^2-2n_y sx+n_y^2}$$

$$(2n_y s-2kb_x)x+(kb_x^2-n_y^2-r_n^2)=2r_n\sqrt{kx^2-2n_y sx+n_y^2}$$

$$ux+v=2r_n\sqrt{kx^2-2n_y sx+n_y^2}$$

$$u^2x^2+2uvx+v^2=4r_n^2(kx^2-2n_y sx+n_y^2)$$

$$u^2x^2+2uvx+v^2=4r_n^2kx^2-8r_n^2n_y sx+4r_n^2n_y^2$$

$$u^2x^2+2uvx+v^2-4r_n^2kx^2+8r_n^2n_y sx-4r_n^2n_y^2=0$$

$$(u^2x^2-4r_n^2kx^2)+(2uvx+8r_n^2n_y sx)+(v^2-4r_n^2n_y^2)=0$$

$$(u^2-4r_n^2k)x^2+(2uv+8r_n^2n_y s)x+(v^2-4r_n^2n_y^2)=0$$

In standard quadratic equation form, $ax^2 + bx + c = 0$,

$$a=u^2-4r_n^2k$$

$$b=2uv+8r_n^2n_y s$$

$$c=v^2-4r_n^2n_y^2$$